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Exciton–longitudinal-optical-phonon problem in quantum wells

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Abstract. A theory of excitons in quantum wells based on the Bethe–Salpeter equation is presented. The energy gap shift, polaronic effective masses of electrons and holes as well as the ground-state exciton binding energy have been calculated as functions of the layer thickness. In the limit of thin layers the interaction of excitons with LO phonons is found to cause a decrease in the exciton binding energy of the order of 18%.

1. Introduction

Because of the technological development of molecular beam epitaxy that has made it possible to grow high-quality heterojunctions and superlattices, great interest in the electronic properties of the quasi-two-dimensional electron gas has occurred. It is expected that in such systems the confinement of electrons between two parallel potential barriers is responsible for many new physical properties and effects with practical importance in the fabrication of new devices. The usual materials for the above-mentioned structures are the weakly ionic III–V compounds such as GaAs–$Ga_{1-x}Al_xAs$ heterostructures. For these materials the interaction with optical phonons dominates; so the effects of electron–LO-phonon coupling must be taken into account in the description of optical properties.

Turning our attention to the theoretical situation, we find that two fundamental different models have been used to describe excitons in quantum wells. The first is based on the usual bulk Fröhlich model (Das Sarma and Madhukar 1980, Das Sarma 1983, Degani and Hipolito 1987, Matsuura 1987). According to the Fröhlich model the electron and hole that constitute the exciton are well separated, and therefore interact individually with LO phonons. The second model is based on the assumption that LO phonons are confined to the layer (Trallero Gines and Comas 1988, Rudin and Reinecke 1990). The last assumption leads to the fundamental differences between the electron–phonon interaction in heterostructures and the Fröhlich model, since the z component of the phonon wavevector has a discrete nature. One important consequence of the second approach is that, in the limit, when the well thickness $L \to 0$ the polaronic energy shift and polaronic mass corrections for weak-coupling polarons tend to zero, while the Fröhlich model predicts the correct results for the energy shift and for the mass corrections not only in the three-dimensional (3D) bulk case but also in the two-dimensional (2D) case as well (Guseinov and Seid-Rzaeva 1989). In the present work we shall assume

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only a perfect confinement of electrons and holes in the well, since the condition for completely confined LO phonons is questionable considering the small difference between the lattice constants of the materials in the heterostructures.

Strictly speaking, the above two approaches are not entirely correct pictures, since the more detailed theory has to be based on the assumption that well defined elementary excitations (polaritons) formed by coupling of excitons, phonons and photons exist in quantum wells. In the bulk case the corresponding theory has already been proposed (Koinov 1990). According to this theory the energy spectrum of excitons in quantum wells manifests itself as poles in the frequency plane of the exciton Green function and, therefore, they can be obtained by means of the corresponding Bethe–Salpeter equation. This equation has the same form as in the bulk case, but in the quantum wells the following additional difficulties occur.

(i) The symmetry is broken in the z direction.

(ii) The electrons and holes are confined in the quantum wells.

(iii) The forms of the photon and phonon Green functions in quantum wells are more complicated than in the bulk case owing to the different lattice and dielectric constants of the layers.

The above-mentioned peculiarities lead to serious difficulties in any attempt to formulate a detailed theory of excitons in quantum wells which result from the different lattice and dielectric constants of the layers. In what follows we have neglected these effects because of the nearly equal lattice and dielectric constants of GaAs and $Ga_{1-x}Al_xAs$.

The paper is organized as follows. In section 2 an effective Bethe–Salpeter equation for the exciton wavefunction is derived. Using this equation the energy gap shift and the polaronic effective masses are calculated in section 3. In section 4 the effective electron–hole interaction in a quantum well is derived and using a variational method the ground-state energy of excitons in quantum wells for different thicknesses is calculated.

2. Bound-state equation

We are interested in the eigenvalues and corresponding wavefunctions of the Wannier excitons in quantum wells made from direct-gap semiconductors with non-degenerate and isotropic bands. With the perfect-confinement approximation for electrons and holes the exciton wavefunction can be written as

$$\psi^{nQ}(\rho, z_e, z_h) = \sum_{\lambda,\mu} \varphi_\lambda(z_e)\varphi_\mu(z_h)\Phi^{nQ}_{\lambda\mu}(\rho) \tag{1a}$$

where

$$\varphi_\lambda(z) = (2/L)^{1/2} \sin(\lambda\pi z/L). \tag{1b}$$

In the above equations, n denotes the nth eigenstate of the exciton with 2D wavevector Q in the quantum well with a thickness L; $\rho = (x_e, y_e, 0) - (x_h, y_h, 0)$ where $r_e = (x_e, y_e, z_e)$ and $r_h = (x_h, y_h, z_h)$ are the coordinates of electrons and holes; $\lambda, \mu = 1, 2, \ldots$ denote the quantum number of the states in the infinitely deep wells. As was mentioned above, the exciton spectra $\omega_n(Q)$ can be obtained by searching for the poles of the two-particle electron–hole Green function in the complex energy plane. A suitable algorithm for locating these poles by reducing the Bethe–Salpeter integral equation for the exciton–

four-time-variable Green function to an effective eigenvalue equation has been proposed in a previous paper (Koinov 1990). Taking into account the broken symmetry on the z direction and following the above-mentioned algorithm, one can obtain the following equation for the binding energy of the exciton $E_n(Q) = E_g - \hbar\omega_n(Q)$:

$$\left[E_n(Q) + E_c(k+Q,\lambda) + E_v(k,\mu) - \sum_{\lambda\mu}^{c}(k,Q,E_n) - \sum_{\lambda\mu}^{v}(k,Q,E_n)\right]\Phi_{\lambda\mu}^{nQ}(k)$$
$$- \sum_{\xi(\neq\lambda)}\left[\sum_{\lambda\mu\xi}^{c}(k,Q,E_n)\Phi_{\xi\mu}^{nQ}(k)\right] - \sum_{\eta(\neq\mu)}\left[\sum_{\lambda\mu\eta}(k,Q,E_n)\Phi_{\lambda\eta}^{nQ}(k)\right]$$
$$- \sum_{\xi,\eta}\int\frac{d^2p}{(2\pi)^2}I_{\text{eff}}\left(\begin{matrix}\lambda & \xi\\ \mu & \eta\end{matrix}\middle| k,p,Q,E_n\right)\Phi_{\xi\eta}^{nQ}(p) = 0 \tag{2}$$

where $\Phi_{\lambda\mu}^{nQ}(k)$ is the Fourier transform of $\Phi_{\lambda\mu}^{nQ}(\rho)$. In equation (2) we have introduced the following notation:

$$E_c(k,\lambda) = \hbar^2k^2/2m_c + \hbar^2\pi^2\lambda^2/2m_cL^2 \tag{3a}$$

$$E_v(k,\lambda) = \hbar^2k^2/2m_v + \hbar^2\pi^2\lambda^2/2m_vL^2. \tag{3b}$$

Equation (2) is our effective Bethe–Salpeter equation for the exciton wavefunction $\Phi_{\lambda\mu}^{nQ}(k)$. The self-energy corrections Σ^c and Σ^v as well as the effective potential I_{eff} are defined as follows:

$$\sum_{\lambda\mu\nu}^{c}(k,Q,E_n) = \frac{2\pi\hbar\omega_0e^2}{\varepsilon^*}\int\frac{d^2p}{(2\pi)^2}\frac{1}{2|p|}$$
$$\times\sum_{\nu}\frac{F_{\lambda\nu\xi\nu}(L|p|)}{\hbar\omega_0 + E_n(Q) + E_c(p+k+Q,\nu) + E_v(k,\mu)} \tag{4a}$$

$$\sum_{\lambda\mu\eta}^{v}(k,Q,E_n) = \frac{2\pi\hbar\omega_0e^2}{\varepsilon^*}\int\frac{d^2p}{(2\pi)^2}\frac{1}{2|p|}$$
$$\times\sum_{\nu}\frac{F_{\eta\nu\mu\nu}(L|p|)}{\hbar\omega_0 + E_n(Q) + E_c(k+Q,\lambda) + E_v(k-p,\mu)} \tag{4b}$$

$$\sum_{\lambda\mu}^{c}(k,Q,E_n) = \sum_{\lambda\mu\lambda}^{c}(k,Q,E_n) \tag{5a}$$

$$\sum_{\lambda\mu}^{v}(k,Q,E_n) = \sum_{\lambda\mu\mu}^{v}(k,Q,E_n) \tag{5b}$$

$$I_{\text{eff}}\left(\begin{matrix}\lambda & \xi\\ \mu & \eta\end{matrix}\middle| k,p,Q,E_n\right) = (2\pi e^2/|p-k|)F_{\lambda\mu\eta\xi}(L|p-k|)$$
$$\times\left[1/\varepsilon_\infty - \frac{\hbar\omega_0}{2\varepsilon^*}\left(\frac{1}{\hbar\omega_0 + E_n(Q) + E_c(p+Q,\xi) + E_v(k,\mu)}\right.\right.$$
$$\left.\left. + \frac{1}{\hbar\omega_0 + E_n(Q) + E_c(k+Q,\lambda) + E_v(p,\eta)}\right)\right] \tag{6}$$

where ε_∞ and ε_0 are the optical and static dielectric constants, $\varepsilon^{*-1} = \varepsilon_\infty^{-1} - \varepsilon_0^{-1}$ and ω_0 is the longitudinal optical phonon frequency. The function $F_{\lambda\mu\eta\xi}$ is defined as follows:

$$F_{\lambda\mu\eta\xi}(L|p|) = \int_0^L dz_1 \int_0^L dz_2 \exp(-|p||z_1 - z_2|)\, \varphi_\lambda(z_1)\varphi_\mu(z_2)\varphi_\eta(z_2)\varphi_\xi(z_1). \tag{7}$$

3. Band-gap renormalization and polaron effective masses

It is well known that owing to the interaction between the particles (electrons and holes) and the ions an extra mass $\Delta m_{c,v} = m^*_{c,v} - m_{c,v}$ arises and, therefore, the polaron effects make the particle appear to be heavier than the band masses. The simplest way to obtain polaron effective masses of electrons and holes is to neglect the excitonic effects. In this case, one should assume the effective potential I_{eff} to be equal to zero, so that the Bethe–Salpeter equation (2) is reduced to an equation for the electron–hole pair formed by λ and μ subbands. As can be seen from equation (2), the interaction with phonons causes a subband mixing, i.e. the energy of the (λ, μ)-electron–hole pair depends not only on the subbands $E_c(k, \lambda)$ and $E_v(k, \mu)$ but also on the rest of the subbands. In the lowest-order approximation we neglect the mixing between subbands by taking $\Sigma^c_{\lambda\mu\xi}$ and $\Sigma^v_{\lambda\mu\eta}$ in equation (2) to be equal to zero. In this approximation, one can write for the energy of electron–hole pair, formed by the lowest conduction ($\lambda = 1$) and valence ($\mu = 1$) subbands,

$$E^{e-h}_{11}(k) = E^{\text{eff}}_g + \hbar^2k^2/2m_c + \hbar^2k^2/2m_v + \Sigma^c_{11}(k, Q = 0, E^{e-h}_{11}) + \Sigma^v_{11}(k, Q = 0, E^{e-h}_{11}) \tag{8}$$

where we have introduced an effective band gap $E^{\text{eff}}_g = E_g + E^c_L + E^v_L$; $E^{c,v}_L = \hbar^2\pi^2/2m_{c,v}L^2$. From equation (8), one can derive the energy gap shift ΔE_g and the corresponding polaronic masses by expanding Σ^c_{11} and Σ^v_{11} in a power series in k. Thus, we obtain the following equation for ΔE_g:

$$\frac{\Delta E_g}{\hbar\omega_0} = \alpha_c \sum_{\lambda=1,2,\ldots} \int dz \frac{F_{1\lambda1\lambda}(zL/r^c_p)}{z^2 + 1 + \Delta E_g/\hbar\omega_0 + \pi^2(\lambda^2 - 1)(r^c_p/L)^2} + \alpha_v \sum_{\lambda=1,2,\ldots} \int dz \frac{F_{1\lambda1\lambda}(zL/r^v_p)}{z^2 + 1 + \Delta E_g/\hbar\omega_0 + \pi^2(\lambda^2 - 1)(r^v_p/L)^2}. \tag{9}$$

Here the electron and hole polaron radii r^c_p and r^v_p and the electron–phonon and hole–phonon coupling constants α_c and α_v are defined as usual: $r^{c,v}_p = (\hbar/2m_{c,v}\omega_0)^{1/2}$; $\alpha_{c,v} = (m_{c,v}e^4/2\hbar^3\omega_0\varepsilon^{*2})^{1/2}$, and the following functions are introduced:

$$F_{1111}(z) = (3z^2 + 8\pi^2)/z(z^2 + 4\pi^2) - 32\pi^4[1 - \exp(-z)]/z^2(z^2 + 4\pi^2)^2 \tag{10a}$$

$$F_{1\lambda1\lambda}(z) = 2z[z^2 + \pi^2(\lambda^2 + 1)]/[z^2 + \pi^2(\lambda - 1)^2][z^2 + \pi^2(\lambda + 1)^2] - 32\pi^4\lambda^2z^2[1 - (-1)^{1+\lambda}\exp(-z)]/[z^2 + \pi^2(\lambda - 1)^2]^2[z^2 + \pi^2(\lambda + 1)^2]^2. \tag{10b}$$

The polaronic effective masses for the $\lambda = 1$ and $\mu = 1$ subbands are found to be

$$\frac{m^*_{c,v}}{m_{c,v}} = \left(1 - 2\alpha_{c,v} \sum_{\lambda=1,2,\ldots} \int dz \frac{z^2 F_{1\lambda1\lambda}(zL/r^{c,v}_p)}{[z^2 + 1 + \Delta E_g/\hbar\omega_0 + \pi^2(\lambda^2 - 1)(r^{c,v}_p/L)^2]^3}\right)^{-1}. \tag{11}$$

In the limit $L \to 0$ the following two equations hold:

$$(\Delta E_g/\hbar\omega_0) = (\alpha_c + \alpha_v)\pi/2(1 + \Delta E_g/\hbar\omega_0)^{1/2}$$

$$m^*_{c,v}/m_{c,v} = [1 - \pi\alpha_{c,v}/8(1 + \Delta E_g/\hbar\omega_0)^{1/2}]^{-1}.$$

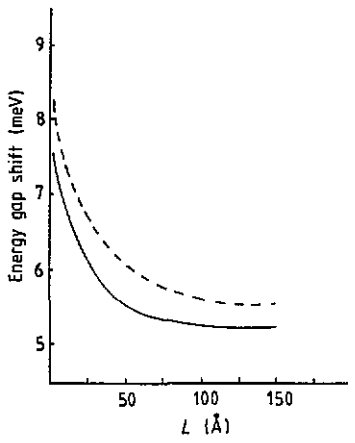

Figure 1. Calculated energy gap shift of a GaAs ($m_c = 0.067m_0$; $m_v = 0.197m_0$; $\alpha_c = 0.0681$; $\alpha_v = 0.092$; $\hbar\omega_0 = 35.2$ meV) quantum well as a function of the quantum-well thickness (——): – – –, obtained by neglecting $\Delta E_g/\hbar\omega_0$ in the right-hand side of equation (9).

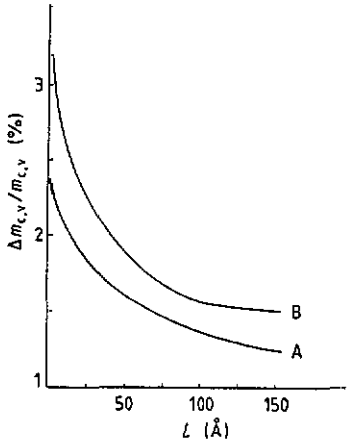

Figure 2. Calculated percentage extra masses of electrons (curve A) and holes (curve B) of a GaAs ($m_c = 0.067m_0$; $m_v = 0.197m_0$; $\alpha_c = 0.0681$; $\alpha_v = 0.092$; $\hbar\omega_0 = 35.2$ meV) quantum well as a function of the quantum-well thickness.

The conventional 2D polaron corrections ΔE_g and $m^*_{c,v}$ can be obtained from the above equations by neglecting $\Delta E_g/\hbar\omega_0$ in their right-hand sides.

Using equations (9)–(11) we have calculated the energy shift ΔE_g and the percentage polaronic extra masses $\Delta m_{c,v}/m_{c,v} = (m^*_{c,v}/m_{c,v} - 1)\%$ for heavy-hole excitons in GaAs-$Ga_{1-x}Al_xAs$. The values of the physical relevant parameters that we have used are $m_c = 0.067m_0$, $m_v = 0.197m_0$, $\alpha_c = 0.0681$, $\alpha_v = 0.092$ and $\hbar\omega_0 = 35.2$ meV (Brum and Bastard 1985). Figures 1 and 2 show the results we have obtained for the energy gap shift ΔE_g and for the percentage extra masses which are plotted as functions of the layer thickness. In the paper by Mori *et al* (1988), $\Delta m_c/m_c$ was found to be about 13% ($m^*_c = 0.076m_0$) for $L = 100$ Å. As can be seen from figure 2 our extra mass correction is too small to achieve agreement between theory and experiment. The discrepancies between calculated and measured masses are probably due to the non-parabolic characteristic of electron band.

4. Effective electron–hole interaction in a quantum well

In section 2 we have obtained an exact state equation where the effective potential I_{eff} depends on k and p separately and is therefore non-local. We now transform the wave equation (2) to coordinate space. The r-space wavefunction $\Psi^{nQ}(\rho, z_e, z_h)$ is defined by equation (1) and the eigenstate equation becomes

$$\left(E_n(Q) - \Delta E_g - \frac{\hbar^2 Q^2}{2M^*} - \frac{\hbar^2}{2\mu^*}\nabla_\rho^2 - \frac{\hbar^2}{2m_c^*}\frac{\partial^2}{\partial z_e^2} - \frac{\hbar^2}{2m_v^*}\frac{\partial^2}{\partial z_h^2}\right)\Psi^{nQ}(\rho, z_e, z_h)$$
$$= \int d\rho' \int_0^L dz'_e \int_0^L dz'_h \, V_{\text{eff}}(\rho, z_e, z_h; \rho, z'_e, z'_h; E_n, Q)$$
$$\times \Psi^{nQ}(\rho', z'_e, z'_h) \tag{12}$$

where $M^* = m_c^* + m_v^*$, $\mu^* = m_c^* m_v^*/M^*$ and the effective potential is defined as follows:

$$V_{\text{eff}}(\rho, z_e, z_h; \rho, z_e', z_h'; E_n, Q)$$
$$= \delta(z_e - z_e')\delta(z_h - z_h')\delta(\rho - \rho')\{e^2/\varepsilon_\infty[\rho^2 - (z_e - z_h)^2]^{1/2}\}$$
$$- V_\Omega(\rho, z_e, z_h; \rho, z_e', z_h'; E_n, Q) \tag{13}$$

$$V_\Omega(\rho, z_e, z_h; \rho, z_e', z_h'; E_n, Q) = \sum_{\lambda,\mu} \varphi_\lambda(z_e)\varphi_\mu(z_h)\varphi_\mu(z_h')\varphi_\lambda(z_e')$$
$$\times \frac{\pi\hbar\omega_0 e^2}{\varepsilon^*} \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2p}{(2\pi)^2} \exp[i(k\rho - p\rho')] \frac{1}{|p-k|}$$
$$\times \left(\frac{\exp[-|p-k||z_e - z_h'|]}{\hbar\omega_0 + E_n(Q) + E_c(p+Q,\lambda) + E_v(k,\mu)} \right.$$
$$\left. + \frac{\exp[-|p-k||z_e' - z_h|]}{\hbar\omega_0 + E_n(Q) + E_c(k+Q,\lambda) + E_v(p,\mu)} \right). \tag{14}$$

One can solve equation (12) by expanding the exciton wavefunction into a basis $R_{n,m}(\rho)$ of the radial functions of a 2D hydrogen atom system (Shinada and Sugano 1966) and diagonalizing the corresponding Hamiltonian matrix. For simplicity we assume the total 2D momentum of the exciton to be equal to zero ($Q = 0$). In this case the wavefunction $\Psi(\rho, z_e, z_h)$ of the lowest exciton state with binding energy $E_0 = \Delta E_g - E_{n=0}(Q=0)$ can be written in the form

$$\Psi(\rho, z_e, z_h) = \sum_{\lambda,\mu} \sum_{n,m} C_{n,m}(\lambda,\mu)\varphi_\lambda(z_e)\varphi_\mu(z_h)R_{n,m}(\rho) \tag{15}$$

where $C_{n,m}(\lambda,\mu)$ satisfies the following equation:

$$\left(E_0 + \frac{\hbar^2\pi^2\lambda^2}{2m_c^* L^2} + \frac{\hbar^2\pi^2\mu^2}{2m_v^* L^2} - E_n^{(2D)}\right) C_{n,m}(\lambda,\mu)$$
$$- \frac{2\pi e^2}{\varepsilon_\infty} \sum_{n_1,m_1} \sum_{\xi,\eta} \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} \frac{F_{\lambda\mu\eta\xi}(L|p-k|) - \delta_{\lambda\xi}\delta_{\mu\eta}}{|p-k|}$$
$$\times R_{n_1,m_1}(p)R_{n,m}(k)C_{n_1,m_1}(\xi,\eta) + \frac{\pi\hbar\omega_0}{\varepsilon^*} \sum_{n_1,m_1} \sum_{\xi,\eta} \int \frac{d^2p}{(2\pi)^2}$$
$$\times \int \frac{d^2k}{(2\pi)^2} \frac{F_{\lambda\mu\eta\xi}(L|p-k|)}{|p-k|} \left(\frac{1}{\hbar\omega_0 + E_0 + E_c(p,\xi) + E_v(k,\mu)} \right.$$
$$\left. + \frac{1}{\hbar\omega_0 + E_0 + E_c(k,\lambda) + E_v(p,\eta)} \right) R_{n_1,m_1}(p)R_{n,m}(k)C_{n_1,m_1}(\xi,\eta) = 0. \tag{16}$$

Here $E_n^{(2D)}$ is the energy of the 2D system, $n = 0, 1, 2, \ldots$ is the principal quantum number and, for a given n, the angular momentum quantum number $m = 0, \pm 1, \pm 2, \ldots$ and $R_{n,m}(k)$ is the Fourier transform of $R_{n,m}(\rho)$.

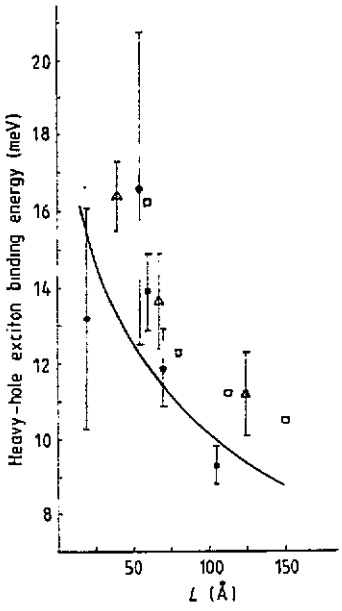

Figure 3. Heavy-hole ground-state exciton binding energy as a function of the GaAs (m_c = $0.067m_0$; m_v = $0.197m_0$; ε_∞ = 10.9; ε_0 = 12.5; $\hbar\omega_0$ = 35.2 meV) quantum-well thickness: ●, experimental results (Rogers *et al* 1986); △, data from the work of Maan *et al* (1984); □, data from the work of Tarucha *et al* (1984).

Our next aim is to find out how the binding energy depends upon the parameters $R_\infty/\hbar\omega_0$, M^*/μ^*, $\varepsilon_\infty/\varepsilon_0$ and L/r_p where $R_\infty = \mu^* e^4/2\varepsilon_\infty^2\hbar^2$ and $r_p = (\hbar/2\mu^*\omega_0)^{1/2}$ are the exciton Rydberg and polaronic radius, respectively. To do this, we take into account only the diagonal terms in the Hamiltonian matrix and then use the variational method with a wavefunction

$$R_{1,0}(\rho) = (\beta^2/2\pi)^{1/2}\exp(-\beta\rho/2). \tag{17}$$

The parameter β can be determined by maximizing the binding energy E_B with respect to β. For the case when the exciton binding energy is small in comparison with the phonon energy, we can replace the last non-local term in equation (16) by a local term, which has the form

$$\frac{2\pi\hbar\omega_0 e^2}{\varepsilon^*}\int\frac{d^2p}{(2\pi)^2}\exp[ip\rho]\frac{F_{1111}(L|p|)}{|p|(\hbar\omega_0 + E_B + \hbar^2p^2/2M^*)}R_{1,0}^2(p). \tag{18}$$

Thus, we obtain the following equation for the excition binding energy:

$$\begin{aligned}\frac{E_B}{\hbar\omega_0} = -4\beta^2 &+ 128\beta^3\left(\frac{R_\infty}{\hbar\omega_0}\right)^{1/2}\int_0^{+\infty}dx\,\frac{F_{1111}(xL/r_p)}{(x^2+16\beta^2)^{3/2}}\\ &- 128\beta^3\left(\frac{R_\infty}{\hbar\omega_0}\right)^{1/2}\frac{M^*}{\mu^*}\left(1-\frac{\varepsilon_\infty}{\varepsilon_0}\right)\\ &\times\int_0^{+\infty}dx\,\frac{F_{1111}(xL/r_p)}{(x^2+16\beta^2)^{3/2}(x^2+M^*/\mu^*)}.\end{aligned} \tag{19}$$

We have numerically maximized the energy expression (19) for the different values of the layer thickness L of the GaAs quantum wells. The values of the physical relevant parameters are the same as in section 3. Figure 3 shows the results that we have obtained.

One can see that our results are in good agreement with the experimental values (Maan *et al* 1984, Tarucha *et al* 1984, Rogers *et al* 1986). The discrepancy between calculated and measured binding energies can be made smaller if one takes into account off-diagonal terms in Hamiltonian matrix in equation (16).

5. Conclusion

Our variational calculations for the energy gap shift, extra polaronic masses and binding energies of excitons in quantum wells show that they decrease monotonically with increasing well thickness. The theory presented above is based on several assumptions which we believe do not affect the essential features of the results.

In our paper we have developed a method based on the Bethe–Salpeter equation for answering the question which naturally arises in the theory of excitons in quantum wells as to whether the polaronic corrections are important, mainly in the limit of thin layers. According to our calculations, we derived polaronic corrections to the exciton binding energy of about 18%.

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